

• Αλκυονίδων στον Cantor.

Έστω  $\alpha_n, n \in \mathbb{N}$  και  $\beta_n, n \in \mathbb{N}$  και  $\lim(\alpha_n - \beta_n) = 0$

$\mathbb{N} \Delta \mathbb{O}$   $\alpha_n \uparrow$  και  $\beta_n \downarrow$

i) Η ακολουθία  $\gamma_n = \beta_n - \alpha_n, n \in \mathbb{N}$  είναι  $\downarrow$

ii) Για την  $\gamma_n, n \in \mathbb{N}$  ισχύει ότι  $\gamma_n \geq 0$

iii)  $\lim \alpha_n = \lim \beta_n, \forall n \in \mathbb{N}$

ΜΕΤ

$$\begin{aligned} \text{i) } \gamma_{n+1} - \gamma_n &= \beta_{n+1} - \alpha_{n+1} - (\beta_n - \alpha_n) = \\ &= \underbrace{\beta_{n+1} - \beta_n}_{(-)} - \underbrace{(\alpha_{n+1} - \alpha_n)}_{(+)} \leq 0 \end{aligned}$$

Άρα,  $\gamma_n, n \in \mathbb{N}$  είναι  $\downarrow$

ii) Θεώρημα  $\gamma_n \geq 0 \Rightarrow \beta_n - \alpha_n \geq 0 \Rightarrow \alpha_n \leq \beta_n, n \in \mathbb{N}$

Ξεχωριστά, ότι  $\lim(\beta_n - \alpha_n) = 0 \Rightarrow \lim \gamma_n = 0$

Άρα, η  $\gamma_n, n \in \mathbb{N}$  αφού φραγμένη και  $\downarrow$

$$\Rightarrow \lim \gamma_n = \inf \{ \gamma_n, n \in \mathbb{N} \} = 0 \Rightarrow$$

$$\Rightarrow \inf \{ \beta_n - \alpha_n, n \in \mathbb{N} \} = 0 \Rightarrow$$

$$\Rightarrow \beta_n - \alpha_n \geq 0 \Rightarrow \alpha_n \leq \beta_n, n \in \mathbb{N}$$

iii) Από το Θεώρημα του Cantor:

$$\alpha_1 \leq \alpha_2 \leq \dots \leq \beta_n \leq \dots \leq \beta_2 \leq \beta_1$$

$\alpha_n, n \in \mathbb{N} \uparrow$  με  $a_n \beta_1$ ,  $\beta_n, n \in \mathbb{N} \downarrow$  με  $\beta_1 \alpha_1$

$$\begin{aligned} \lim \beta_n &= \lim (\beta_n - \alpha_n + \alpha_n) = \lim (\beta_n - \alpha_n) + \lim \alpha_n = \\ &= \lim \alpha_n, n \in \mathbb{N} \end{aligned}$$